四十周年校慶
路德會西門英才中學

數學

趣味四十談

Light of the World

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學而時習之，

不亦說乎？

校長簡加言題

2017 年 10 月

Isn't it pleasurable to
have frequent revision
on all the knowledge learnt?

From Principal Kan
Oct 2017

數學趣味四十談

序言

數學給人的感覺往往是冷冰冰的，但箇中的樂趣，又有多少同學能感受到呢？這本數學趣味四十談，就是希望能從本校一班熱血的數學鍾情者，介紹一些有趣的數學題目給大家嘗嘗。

雖然同學們有時會覺得數學與日常生活格格不入，但數學世界中包含了的邏輯、觀察力、歸納性、創造力及解難能力，正正是數學帶給人們的有趣的禮物。

本小冊子包含了不同類別的數學謎題，解說如何利用各種不同的技巧來解題，當中有些更是同學在答題上常見的錯誤。這是一本實用的工具書，可以幫助同學擴闊思維，學懂不同的解難技巧，其中強調解題的樂趣及須突破固有觀念的問題也不少。

我盼望這些工具能提升同學對學習數學的興趣，並能誘使同學更主動積極地去學習及懂得如何更好地學習有趣的數學。

數學科科主任

何家豪

2017 年 7 月

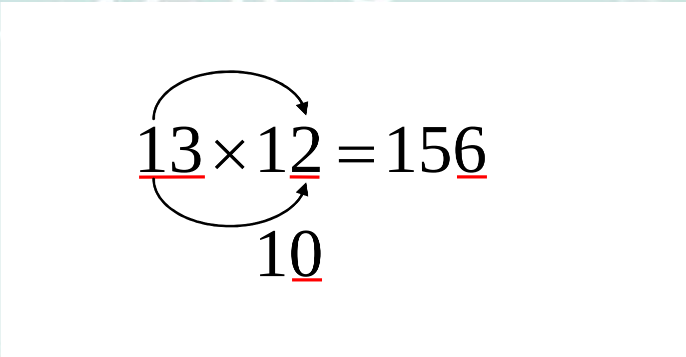
目錄

意想不到的數學

Q1. 印度乘法 (1)

印度乘法是一個快速計算方法，主要是利用乘法分配性質計算，此方法有一個限制，使用時只可應用於十位數為「1」時的題目。例：

Multiplication of India is a fast calculation method, mainly using the multiplication of distribution. The limitation of this method is only being applied to the ten-digit which is "1". For example:



先將「13」加上另一個數的個位，然後抄下數值，再將兩數的個位相乘並加在剛才的數值右方便完成。

“13" is added to another number of units, and then copy the value. After that, multiplied the number of units and added them to the right-handed side of the value which is copied before.

Q2. 印度乘法 (2)

另外的印度乘法比以上的版本能計算更多類型的數學題，可是此方法亦有限制，使用時只可以應用於十位數一樣的題目，如：63×67。用法：先將十位數其中一個加一並相乘，以上題為例，6×7 並將答案抄下，之後將 3×7，再將答案抄寫在剛才的數值右方。

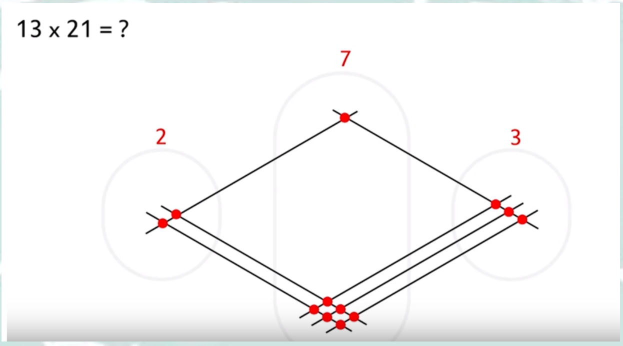
Second version of the Multiplication of India which can calculate more types of math problems, but this method is also limited. The usage can only be applied to the same number of ten-digit, such as 63×67.

Method：First, one of the ten digits plus one and multiply to the other, then copy the result of 6×7, like the above example. Finally, copy the result of 3×7 in the right hand side.

Q3. 日本畫線乘法

日本數學畫線乘法是使用簡單直觀線條圖技術將兩個數相乘，只需要數算線條上的交點數目，並有次序地記下便計算完成。

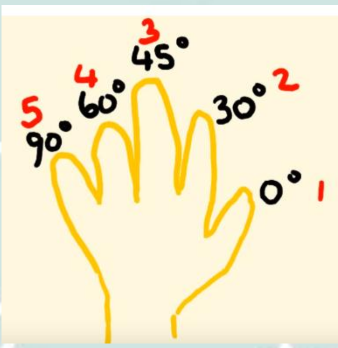
Japanese mathematical line multiplication is the usage of simple and intuitive line diagram technology to multiply the two numbers. It only needs to count the number of points of the intersection by the lines, and write down the calculation in order.



Q4. 三角學的特殊角

當利用三角學的特殊角計算時都比較困難，因為你在使用過程中需要記清楚特殊角的數值，影片中有一個比較簡易的記法。

It is difficult to calculate the special angles of the trigonometry, because you need to remember the value of the special angles in the process.There is a relatively simple notation in the clip.



Q5. 循環小數

$$0.\dot{9} = 1 \ \ ????$$

『跳脫』的數學

Q6. 誤解一

若 (If) $2a = 3b$

則 (then) $a\ ：\ b = 2\ ：\ 3$

以上運算中，哪裡出現錯誤?

Q7. 誤解二

$$2x - x = 0$$
$$2x = x$$
$$\frac{2x}{x} = \frac{x}{x}$$
$$2 = 1$$

以上運算中，哪裡出現錯誤?

答案：

Q6. 誤解一

$$2a = 3b$$
$$\frac{a}{b} = \frac{3}{2}$$
$$a\ ：\ b = 3\ ：2$$

Q7. 誤解二

小學時(In the Primary school):

$$2x - x = 0$$
$$x = 0$$

如果(If)：

$$\frac{2x}{x} = \frac{x}{x}$$
$$\boxed{?}\,x = 0$$
$$\therefore \frac{0}{0} = \frac{0}{0}$$

由於 $\frac{0}{0}$ 沒有定義，所以用此方式計算會出現矛盾。

Using this method will be a contradiction because $\frac{0}{0}$ is undefined.

Q8. 誤解三

因式分解 ≠ 解方程 (不少同學將兩者混淆)
Factorize ≠ Solve Equation (Many students misunderstand it.)

例如: 因式分解 $169y^2 - 25$ (CE1999 Q6b)
Example: Factorize $169y^2 - 25$ (CE1999 Q6b)

錯誤例子: Common mistakes e.g.

$169\ y^2 - 25 = 0$

$(13y - 5)(13y + 5) = 0$

$y = \frac{13}{5}$ 或 $y = -\frac{13}{5}$ 錯誤 Wrong [考評局報告 HKCEE Report]

另外，亦有不少考生回答
On the other hand, many students also answer incorrectly as

$169y^2 - 25 = (13y - 5)^2$ 錯誤 Wrong

應回答: Correct answer is:

$169y^2 - 25 = (13y - 5)(13y + 5)$

Q9. 誤解四

有些學生會把如 8×n×(n–1)×(n–2) ×…×3×2×1 的數式，不適當地寫成 8n！。我們在閱讀的時候，可能會錯誤理解為 (8n)！，即事實上，該數式應以 8(n !) 或 n ! × 8 表示。

Many students show the expression 8×n×(n–1)×(n–2) ×…×3×2×1 incorrectly written as 8n ! . As we read it and misunderstand it be (8n) ! .However, it is better to write as 8(n !) or n ! × 8.

Q10. 特殊值代入法策略 (1)

將特別的數值代入未知數中，再找出與問題相符的答案。

例如: 化簡 $\dfrac{\sin(180° - \theta)}{\cos(360° - \theta)}$

A. $-\cos\theta$
B. $\sin^2\theta$
C. $\tan\theta$
D. $-\tan\theta$

可用簡單的值代入 θ 於算式內，再代 θ 於四個選擇，看哪個選擇與答案相符。如代 θ=22°，可得算式等於 0.4040，A = – 0.9272，B = 0.1403，C = 0.4040，D = – 0.4040。

所以，答案是 C。須注意所代入的值，不應代入：30°、45°、60°等特殊角度，因有機會計算到相同的答案或使分母計到 0。

Strategy (1) Substitution of particular value

Substitute some particular values to the unknown in order to find the correct answer.

Example : Simplify $\dfrac{\sin(180° - \theta)}{\cos(360° - \theta)}$

A. $-\cos\theta$
B. $\sin^2\theta$
C. $\tan\theta$
D. $-\tan\theta$

Use any particular value of θ, e.g. θ=22°,
then we get the value of expression = 0.4040.

Choices A = – 0.9272 B = 0.1403 C = 0.4040 D = –0.4040

Therefore, the answer is C. Make sure that the special angles (30°, 45° and 60°) should not be chosen because it may lead to any same value or the denominator become 0.

Q11. 策略 (2) 驗證法

將所有選擇答案 A 至 D 代入題目中，從而找出與題目完全匹配的選擇。

例如: 解 $x(x-6)=x$ [CE2004 Q7]

A. $x = 6$
B. $x = 7$
C. $x = 0$ 或 $x = 6$
D. $x = 0$ 或 $x = 7$

代 $x = 6$，兩邊不相等，故刪去 A 和 C。剩下 B 和 D 中都有 $x = 7$ 的選擇，所不同的是 D 有 $x = 0$ 的選擇，故嘗試代入 $x = 0$。由於代入 $x = 0$ 後，左右兩邊相等，所以，正確答案是 D。

Strategy (2) Substitution of choices A to D into the question

Substitute all choices A to D into the question in order to find the correct answer.

Example: Solve $x(x-6)=x$ [CE2004 Q7]

A. $x = 6$
B. $x = 7$
C. $x = 0$ or $x = 6$
D. $x = 0$ or $x = 7$

Substitute $x = 6$, the values of two sides are not equal, so choices A and C are rejected. Except $x = 7$ for the choices B and D, it only different from $x = 0$. After it substitutes $x = 0$, the values of two sides are equal. Therefore, the answer is D.

數學急轉彎

Q12. 數學謎題

一個數真有趣，
自己加自己，
自己減自己，
自己乘自己，
自己除自己，
所得結果加一起，
整整八十一。
請你猜猜看，這個數是幾？

There is a funny number x：

A = x plus x

B = x minus x

C = x times x

D = x is divided by x

Given that the sum of A, B, C and D is 81

What is the value of x?

Q13. 如何亮起 12 盞燈？

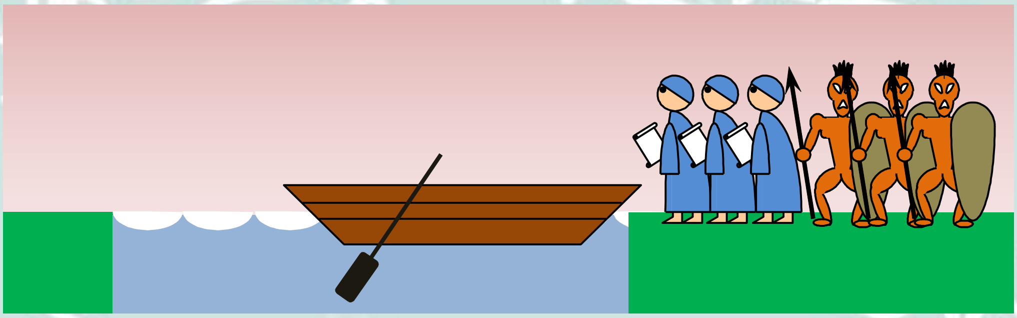
現有 12 盞燈，起初時都是關掉的。每盞燈均有一個按鈕，每次按下按鈕會使關掉的燈亮起，或使亮起的燈關掉。現容許每次按下剛好 5 個不同的按鈕，則最少經過幾次才可使所有燈亮起？

There are 12 lamps, initially all off, each of which comes with a switch. When a switch is pressed, a lamp which is off will be turned on, and a lamp which is on will be turned off. Now one is allowed to press exactly 5 different switches in each round. What is the minimum number of rounds needed so that all lamps will be turned on?

Q14. 傳道士和食人族

3 個傳道士和 3 個食人族土人要渡船到湖的對岸，而船最多容納 2 人。但當一邊的土人多於傳道士時，土人會吞掉傳道士。你能協助他們過河嗎?

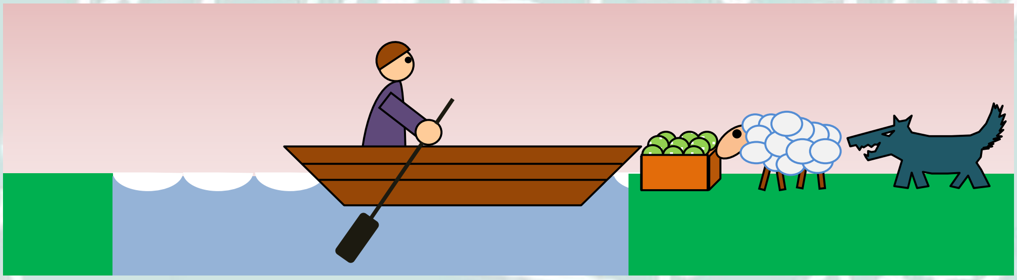
3 preachers and 3 cannibals want to ferry to the other side of the lake, and the boat can accommodate up to 2 people. When the side of the cannibals more than preachers, the cannibals will kill the preacher. Can you help them cross the river?



Q15. 狼、羊與菜

船夫如何將狼、羊和一籃菜送到湖的對岸，而船最多只能運送一款貨物。當船夫不在時，狼將吃掉羊；羊會吃掉菜。你能協助船夫將狼、羊和一籃菜送過河嗎?

A boatman wants to transports a wolf, a sheep and a basket of vegetables to the other side of the river, the ship can only transport up to one loan. When the boatman is not there, the wolf will eat the sheep, and the sheep will eat vegetables. Can you help the boatman transports a wolf, a sheep and a basket of vegetables to the other side of the river?



Q16.【＋、–、×、÷、( )】的 4 數學世界

下圖是 4 的數列。你能適當地放入運算符號 【 +、–、×、÷、( )】，把它們變成右邊的答案嗎？

4	4	4	4	= 0
4	4	4	4	= 1
4	4	4	4	= 2
4	4	4	4	= 3
4	4	4	4	= 4
4	4	4	4	= 5
4	4	4	4	= 6
4	4	4	4	= 7
4	4	4	4	= 8
4	4	4	4	= 9

Q17.【＋、–、×、÷、( )】的 2 數學世界

這裏盡是 2 的系列, 但是如果適當地放入四則運算符號(+-x÷)， 則可以得出一個順序的答案, 0, 1, ..., 6,真是有趣極了!你現在就試試吧!

2	2	2	2	= 0
2	2	2	2	= 1
2	2	2	2	= 2
2	2	2	2	= 3
2	2	2	2	= 4
2	2	2	2	= 5
2	2	2	2	= 6

答案：

Q12. 數學謎題

A = $x+x$　B = $x-x$　C = $x\times x$　D = $x\div x$

A + B + C + D = 81

$(x+x) + (x-x) + (x\times x) + (x\div x) = 81$

$$2x+x^2+1=81$$

$$x = 8 \text{ 或 } x = -10$$

Q13. 如何亮起 12 盞燈？

假設所有燈在 n 步後會全部開，亦代表我們按了按鈕的總次數。留意每一盞燈都會於按單數次數的按鈕而改變亮起或關掉。因有 12 盞燈，所以總按按鈕的次數會是雙數，這亦代表了 n 是偶數。很明顯地，n 不等於 2，而大部份的燈亦會在第二步時已亮起。另外，我們能亮起所有盞燈的步驟如下：

步驟 1 — 按鈕 1, 2, 3, 4, 5
步驟 2 — 按鈕 6, 7, 8, 9, 10
步驟 3 — 按鈕 7, 8, 9, 10, 11
步驟 4 — 按鈕 7, 8, 9, 10, 12

所以答案是 4

Suppose all lamps are turned on after n rounds. Then we have pressed the switches times in total. Note that each lamp should change state for an odd number of times. As there are 12 lamps, the total number of times the lamps have changed state should be an even number. This forces n to be even.
Clearly n is not 2, since at most lamps can be turned on in 2 rounds. On the other hand we can turn on all lamps in 4 rounds as follows:

Round 1 — Press switches 1, 2, 3, 4, 5
Round 2 — Press switches 6, 7, 8, 9, 10
Round 3 — Press switches 7, 8, 9, 10, 11
Round 4 — Press switches 7, 8, 9, 10, 12

It follows that the answer is 4.

Q14. 過河題目

第 1 步：兩土人乘船過河，一土人駕船回來 (右邊餘：3 傳、2 土)

第 2 步：兩土人乘船過河，一土人駕船回來 (右邊餘：3 傳、1 土)

第 3 步：二傳道士乘船過河，一土人及一傳道士駕船回來(右邊餘：2 傳、2 土)

第 4 步：兩傳道士乘船過河，一土人駕船回來 (右邊餘：0 傳、3 土)

第 5 步：兩土人乘船過河，一土人駕船回來 (右邊餘：0 傳、2 土)

第 6 步：兩土人乘船過河 (右邊餘：0 傳，0 土)

Step 1：2 cannibals ferry to the other side, then 1 ferry back.

Step 2：2 cannibals ferry to the other side, then 1 ferry back.

Step 3：2 preachers ferry to the other side, then 1 preacher and 1 cannibal ferry back

Step 4：2 preachers ferry to the other side, then 1 cannibal ferry back.

Step 5：2 cannibals ferry to the other side, then 1 cannibal ferry back.

Step 6：2 cannibals ferry to the other side.

Q15. 狼、羊與菜

第一步：船夫先送羊到對岸 (右邊餘：菜和狼)

第二步：船夫再送走菜及送回羊

第三步：船夫再送走狼 (右邊餘：羊)

第四步：船夫最後送走羊

Step 1： The boatman transports the sheep to the other side.

Step 2： The boatman transports the basket of vegetables to the other side, and transports the sheep back.

Step 3： The boatman transports the wolf to the other side.

Step 4： The boatman transports the sheep to the other side.

Q16.【 +、–、×、÷、( )】的 4 數學世界

4	–	4	+	4	–	4	= 0
4	–	4	+	4	÷	4	= 1
4	÷	4	+	4	÷	4	= 2
(4	+	4	+	4)	÷	4	= 3
(4	–	4)	×	4	+	4	= 4
(4	×	4	+	4)	÷	4	= 5
(4	+	4)	÷	4	+	4	= 6
4	+	4	–	4	÷	4	= 7
(4	×	4)	÷	4	+	4	= 8
4	+	4	+	4	÷	4	= 9

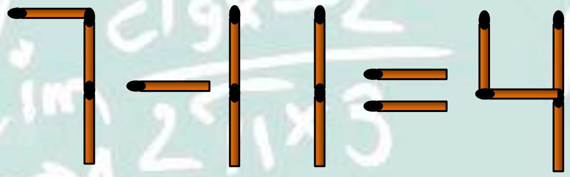
Q17.【 +、–、×、÷、( )】的 2 數學世界

2	+	2	–	2	–	2	= 0
(2	÷	2)	×	(2	÷	2)	= 1
(2	÷	2)	+	(2	÷	2)	= 2
(2	+	2	+	2)	÷	2	= 3
2	×	2	÷	2	+	2	= 4
2	+	2	+	(2	÷	2)	= 5
(2	×	2	×	2)	–	2	= 6

乾坤大挪移

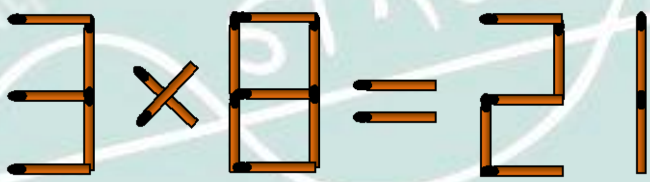
Q18. 移動火柴 (1)

請移動一根火柴使以下算式成立：



Q19. 移動火柴 (2)

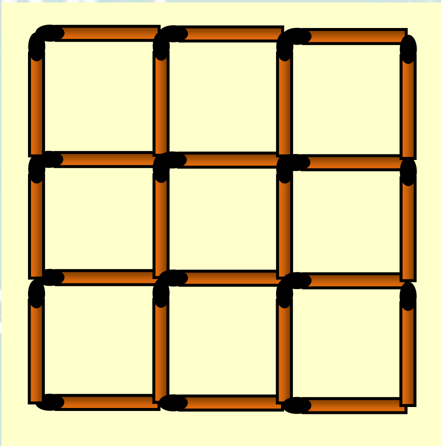
請移動一根火柴使以下算式成立：



Q20. 移動火柴 (3)

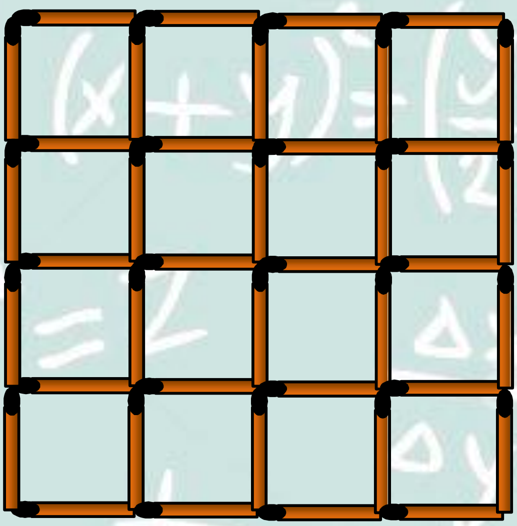
依指示拿走指定數量之火柴，令到圖中大或小的四方形數目和指示一致。

i) 拿走 8 支火柴，餘下四個正方形。
ii) 拿走 4 支火柴，餘下五個正方形。
iii) 拿走 3 支火柴，餘下六個正方形。
iv) 拿走 2 支火柴，餘下七個正方形。
v) 拿走 4 支火柴，餘下七個正方形。



Q21. 移動火柴 (4)

這是一個由 40 根火柴組成的格子，大小正方形共有 30 個,問題是從中取走 9 根火柴棒,而使其中　有一個正方形，那應該如何取法?



Q22. 移動火柴 (5)

5 枝相同長度的火柴，互相交叉相疊，不准折斷，最多有多少個交叉點？

Q23. 移動火柴 (6)

4 枝長度 2 m 的火柴及 4 枝長度 1 m 的火柴，互相交叉相疊組合，不准折斷，如何剛好組成 3 個大少形狀一樣的正方形？

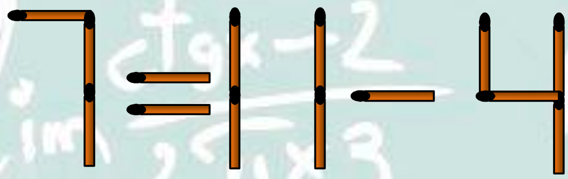
Q24. 移動火柴 (7)

六枝相同長度的火柴，互相交叉相疊組合，不准折斷，如何剛好組成 4 個大少形狀一樣的三角形？

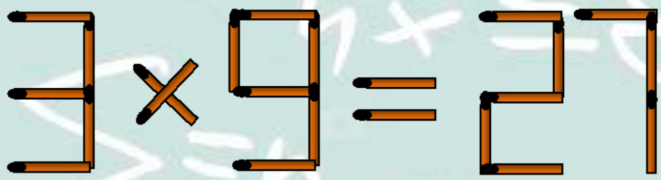
乾坤大挪移

答案

Q18. 移動火柴 (1)

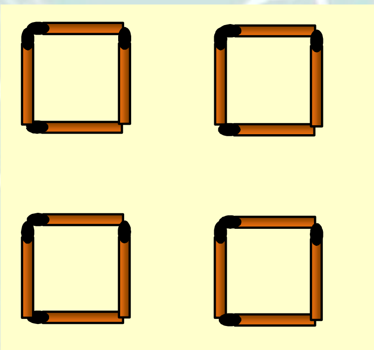


Q19. 移動火柴 (2)

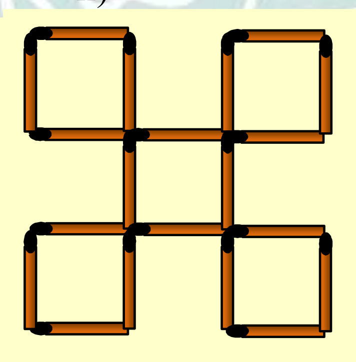


Q20. 移動火柴 (3)

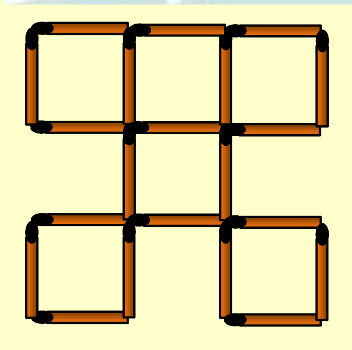
i)



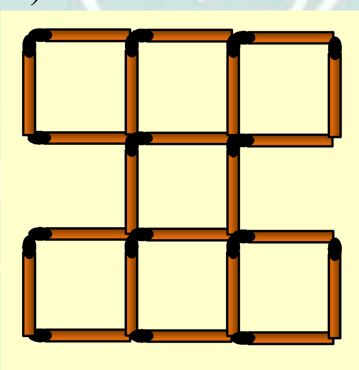
ii)



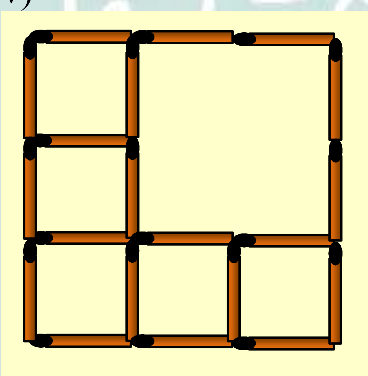
iii)



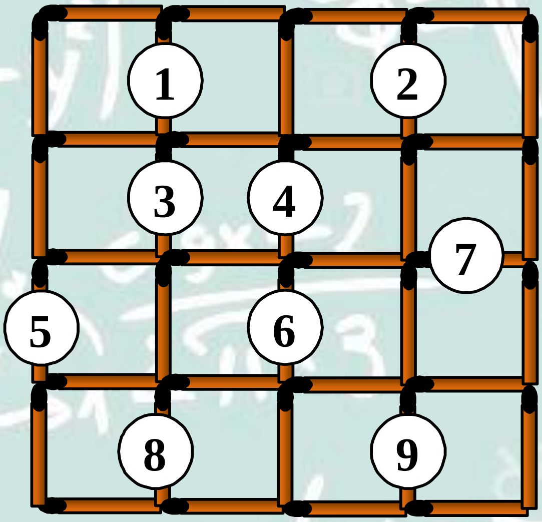
iv)



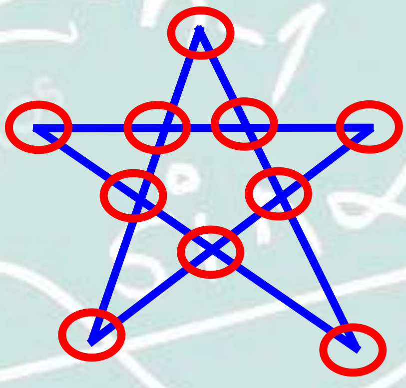
v)



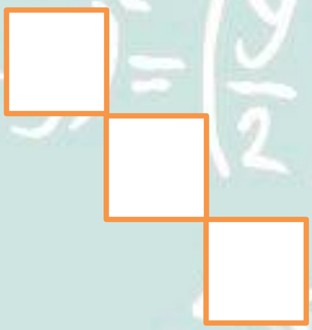
Q21. 移動火柴 (4)



Q22. 移動火柴 (5)



Q23. 移動火柴 (6)



Q24. 移動火柴(7)

正四面體

歷史與數學

由 1 加至 100 的技巧

以下是有關著名數學家高斯的故事。高斯小時候有個懶惰的老師。那老師為了讓學生保持忙碌，使自己得以偷懶，竟讓學生計算由 1 加至 100 的和。

然而，小時候高斯很快就得出答案是 5050。老師以為高斯作弊，但事實並非如此。高斯發現以下公式去求出答案，以避免對一般學生而言十分繁瑣的運算。

由 1 加到 n 之和 $= \dfrac{n(1+n)}{2}$

由 1 加到 100 之和 $= \dfrac{100(1+100)}{2} = (50)(101) = 5050$

Q25. 問題：求 $2+4+6+8+...+200$ 之和的值。

Techniques for Adding the Numbers 1 to 100

There's a popular story that Gauss, mathematician extraordinaire, had a lazy teacher. The teacher wanted to keep the kids busy so he could take a nap; he asked the class to add the numbers 1 to 100.

Gauss approached with his answer: 5050. So soon? The teacher suspected a cheat, but no. Manual addition was for normal students, and Gauss found a formula to sidestep the problem:

Total sum from 1 to n $= \dfrac{n(1+n)}{2}$

Total sum from 1 to 100 $= \dfrac{100(1+100)}{2} = (50)(101) = 5050$

數學的欣賞：第一次數學危機

1. 畢氏學派的信念

公元前 520 年左右，畢達哥拉斯在古希臘創立了一個研究宗教和哲學的團體，稱為「畢氏學派」。這個學派很重視數學，因為他們認為宇宙萬物的基礎都是數。然而，他們所認識的數，只是整數和分數。換言之，他們只認識有理數。據此，他們建立和證明一系列的數學定理。

2. 信念的動搖

後來，他們發現當正方形的邊長為 1 時，其對角線的長度為 $\sqrt{2}$，而 $\sqrt{2}$ 卻無法化為兩個整數的比！這一發現動搖了他們一向的信念，使他們所建立的數學定理失去了依據，在學派內產生極大的震撼，數學史上稱之為「第一次數學危機」。畢氏學派雖然發現了原有信念的謬誤，但他們仍拒絕承認像 $\sqrt{2}$ 等這些平方根是數，並嚴禁門徒向外人透露這個發現。據　，有一位名叫希帕蘇斯（Hippasus）的門徒，因為洩露了這個「祕密」而被投進大海！

3. 危機的解決

大約 200 年後，古希臘的歐多克索斯(Eudoxus，約公元前 400 年至 347 年）創立了比例的新理論，使畢氏學派的許多數學理論可以再次確立，化解了這一次數學危機。

Q26. 問題：已知 $\sqrt{2}$ 不能以 $\frac{a}{b}$ 來表達 (a 及 b 為整數)，試舉出三個數如 $\sqrt{2}$ 般不能以 $\frac{a}{b}$ 來表達。

The Beauty of Mathematics: The First Crisis of Mathematics

1. Beliefs of the Pythagoreans

Around 520 BC, Pythagoras established a religious and philosophical organization called the 'Pythagorean school' in Greece. The followers placed a high importance on Mathematics and believed that the foundations for all things in the universe are numbers. However, the numbers they knew were only integers and fractions. In other words, they knew only the rational numbers. With this fundamental belief, they established and proved a series of mathematical theorems.

2. Shaking of the Beliefs

Later, the Pythagoreans discovered that for a square of side 1, the length of the diagonal is $\sqrt{2}$. However, $\sqrt{2}$ cannot be expressed as a ratio of two integers! This discovery shook violently their beliefs, and the mathematical theorems established so far lost the basic idea of relying solely on integers. This event is called 'the first crisis of Mathematics' in the history of Mathematics.

Although the Pythagoreans realized the error in their original beliefs, they still refused to accept that square roots like $\sqrt{2}$ were numbers at all. Further, they were prohibited from disclosing this discovery. It was said that a follower called Hippasus was thrown into the sea just because he had disclosed this 'secret' to other people!

3. Solution to the Crisis

About 200 years later, the ancient Greek mathematician Eudoxus (~ 400 – 347 BC) proposed a new theory about proportion so that many mathematical results previously found by the Pythagoreans could be established again. This solved 'the first crisis of Mathematics'.

Q26. Given that $\sqrt{2}$ cannot be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers, suggest 3 other numbers like $\sqrt{2}$ and cannot be expressed in the form $\frac{a}{b}$.

π的近似值

圓周率π(即圓形的圓周與直徑之比)在數學上是一個很重要的數值。為了求出π的值，許多數學家付出了不少努力和心血，他們明白到π真確值不可求，於是利用不同的估算方法，希望求得π最準確的近似值。

1. 中國魏晉(公元 220-420)時，劉徽利用「割圓術」，將正多邊形的邊數逐漸增加去逼近一個圓。 他計算了正 6 邊形、正 12 邊形、正 24 邊形、…、以至正 96 邊形的周界，求得π的值在 3.141 024 與 3.142 704 之間。

2. 二百多年後，南北朝的祖沖之求得 π 的值在 3.141 592 6 與 3.141 592 7 之間。雖然他採用甚麼方法已無法考證，但估計他 也是採用劉徽的割圓術，並計算至正 24 576 邊形的周界。

3. 十七世紀德國數學家范瑟朗亦採用類似中國數學家的方法去求π的值。他研究一個2^{62}條邊的正多邊形，求出的 π 值準確至 35 位小數。在超級電腦的輔助下，我們現在已能夠計算出π的值至數兆位小數。

Q27. 求π的值，並準確至 11 位有效數字。

The Approximate Value of π

π is the ratio of the circumference of a circle to its diameter. It is an important numerical value in Mathematics. Many mathematicians have worked very hard to calculate the value of **π**. They understood that the exact value of **π** cannot be obtained and tried to use different methods to find out a more accurate estimated value.

1. In the Wei Jin period of China (AD 220-420), the Chinese mathematician Liu Hui (劉徽) invented a method of calculating **π** by increasing the sides of successive regular polygons to approximate a circle. As shown in the figure, he used the perimeters of regular polygons of different sides as the circumference of a circle to calculate the value of it. He concluded that the value of π lies between 3.141 024 and 3.142 704.
2. About 200 years after Liu Hui, Zu Chong Zhi (祖沖之), in the Northern and Southern Dynasties concluded that the value of **π** lies between 3.141 592 6 and 3.141 592 7.
3. In the 17th century, German mathematician Ludolph van Ceulen estimated the value of **π** by using a method similar to those of the Chinese mathematicians. By studying a 2^{62}-sided regular polygon, he obtained the value of **π** correct to 35 decimal places. Nowadays, we can obtain the value of **π** correct to trillions descimal places by supper computer.

Q27. Find the value of π, and correct to the 11 significant figures.

歐幾里得與《幾何原本》

在二千多年前，古希臘人已累積了很多零碎和不成系統的幾何知識。後來，歐幾里得（Euclid，約公元前 330 年-275 年）竭盡心力把當時已知的幾何知識整理成一套共 13 冊的著作---《幾何原本》。 這本巨著不但包含了用演繹法去建構的幾何知識體系，而且還包含其他數學領域的重要概念，例如數論、比例和立體幾何等。

在幾何命題的演繹推理過程中，需要應用公理、定義及定理，以下 我們會逐一解釋這些名詞的含義。

1. 公理： 一些被人公認為正確的而又無須證明的命題稱為公理。
2. 定義： 對一些圖形、術語作出的規定稱為定義。
3. 定理： 由公理和定義推論出的真命題稱為定理。所得定理可用作推論更多定理。

和許多偉大的數學家一樣，歐幾里得主張學習要循序漸進，專心致志和刻苦鑽研，認為求學問是無平坦道路的。

Q28. 寫出歐幾里得平面幾何的五條公理。

Euclid and Elements

Over 2 000 years ago, the ancient Greeks had accumulated many geometric facts, but these facts were not organized in a systematic and orderly way until the famous mathematician Euclid did so, somewhere around 330 BC to 275 BC. Euclid did a comprehensive study of geometry in his 13-volume work, Elements. Apart from establishing the geometry system using the deductive approach, this big book also included important concepts in other areas of mathematics, such as number theory, ratios and proportions, spatial geometry, etc.

In the process of proving statements in geometry using the deductive approach, we need to use axioms, definitions and theorems. The following are the meanings of these terms:

1. **Axiom**: Some statements which are well-accepted facts so that proofs are not needed are called axioms.
2. **Definition**: The statements about the precise meanings of some indefinitely terms and figures are called definitions.
3. **Theorem**: The true statements deduced from axioms and definitions are called theorems. The theorems obtained can be used to deduce more theorems.

Like many great mathematicians, Euclid believed that people should learn things step by step and have a dedicated and hard-working attitude towards the pursuit of knowledge. He also thought that the road to pursuit of knowledge is not smooth.

Q28. Write down the 5 axioms of Euclidean geometry.

歷史與數學

答案:

Q25. 由 1 加至 100 的技巧

$$\begin{aligned}&2+4+6+8+...+200\\&=2(1+2+3+4+...+100)\\&=2(5050)\\&=10100\end{aligned}$$

Q26. 數學的欣賞：第一次數學危機

$\sqrt{3}$, π , $\sqrt{300}$

Q27. π 的近似值

3.1415926536

Q28. 歐幾里得與《幾何原本》

公理 1：任意一點到另外任意一點可以畫直線。
公理 2：一條有限線段可以繼續延長。
公理 3：以任意點為心及任意的距離可以畫圓。
公理 4：凡直角都彼此相等。
公理 5：同平面內一條直線和另外兩條直線相交。若在某一側的兩個內角和小於兩直角的和，則這兩條直線經無限延長後在這一側相交。

1. To draw a straight line from any point to any point.
2. To produce [extend] a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance [radius].
4. That all right angles are equal to one another."
5. *The parallel postulate*: That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

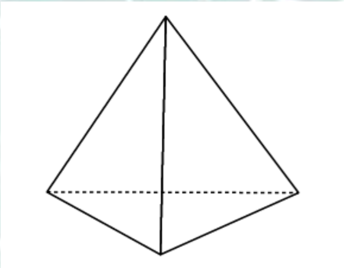
Q29. 播放音樂

某餐廳在下午一時開始重複播放音樂，CD 內有 5 首歌每首長 3 分鐘。周小姐下午一時至二時之間在餐廳內停留 3 分鐘。問周小姐聽到 CD 內第一首歌和第二首歌的概率分別是多少？

The owner starts to play a CD album in a restaurant continuously at 1pm. There are altogether 5 songs in the album and each song lasts for 3 mins. Miss Chow would spend 3 mins in the restaurant between 1pm to 2pm. What is the probability that Miss Chow would hear the first two songs respectively?

Q30. 尤拉公式 (Euler's Formula)

F + V– E = 2 F = 面(Face) V = 頂點(Vertex) E = 邊(Edge)



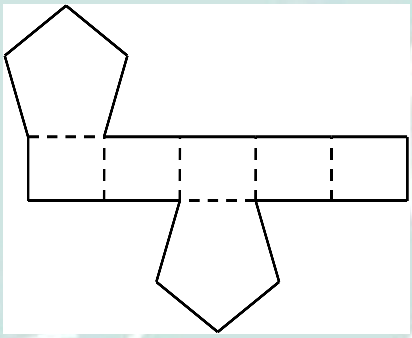
例子 (example) 1

F = 4

V = 4

E = 6

F + V – E = 4 + 4– 6 = 2



左圖是一個多面體的摺紙圖樣。求這多面體的 F、V 及 E 的數目。

The figure shows the nets polyhedra. Find the number of faces (F), number of vertices (V) and number of edges (E) of these polyhedra.

Q31. 殘殺戰俘與死裡逃生

這是一個古老的傳說：有 64 名戰士被敵人俘虜了。敵人命令他們排成一個圓圈，編上號碼：1，2，3 …… 64。敵人把 1 號殺了，又把 3 號殺了，他們是隔一個殺一個這樣轉著圈殺。最後剩下一個人，這個人就是約瑟夫斯。請問約瑟夫斯是多少號？這就是「約瑟夫斯問題」。

There is an old legend: 64 soldiers were captured by their enemies and they were told to form a circle. Each of them was given a number from 1 to 64. The enemies killed all the war prisoners with odd numbers, that is, 1, 3, 5 and so on in the circle. The last one who was not killed was Josephus. What number did Josephus get? This is called the "Josephus Problem".

河內塔

河內塔（中國大陸：漢諾塔， 台灣：河內塔）是根據一個傳說形成的數學問題：

傳說印度某間寺院有三根柱子，上串 64 個金盤。 寺院裏的僧侶依照一個古老的預言，跟從以下所述規則將這些盤子移動到另一根柱子：

1. 每次只能移動一個圓盤；
2. 大盤不能疊在小盤上面。

預言說當這些盤子移動完畢，世界就會滅亡。 這個傳說叫做梵天寺之塔問題（Tower of Brahma puzzle）。 但不知道是法國的數學家盧卡斯自創的這個傳說，還是他受他人啟發。

將 64 個金盤動到另一根柱子，最少需移動$2^{64}-1$次。 即如果一秒鐘能移動一塊金盤，仍將需 5845.54 億年。 目前按照宇宙大爆炸理論的推測，宇宙的年齡僅為 137 億年。

這個傳說有若干變體：寺院換成修道院、僧侶換成修士等等。 寺院的地點眾說紛紜，其中一說是位於越南的河內 ，所以被命名為「河內塔」。 另外亦有「金盤是創世時所造」、「僧侶們每天移動一盤」之類的背景設定。

佛教中確實有「浮屠」（ 塔 ）這種建築；有些浮屠亦遵守上述規則而建。「河內塔」一名可能是由中南半島在殖民時期傳入歐洲的。

Q32： 最少要移動多少次才能將 3 個金盤移動到另一根柱子？

提示：設三根柱子為 A 柱 B 柱 C 柱，可將圓盤臨時置於 B 柱，也可將從 A 柱移出的圓盤重新移回 A 柱，但都必須遵循上述兩條規則。

Hanoi Tower (Mainland China: **Hanoi Tower** , Taiwan: **Hanoi Tower**)
is based on a legend to form the mathematical problem:

Legend of a temple in India has three pillars, on the string of 64 gold plate. The monks in the monasteries moved the plates in accordance with an ancient prophecy, and predicted that when the plates were moved, the world would perish. This legend is called the Tower of Brahma puzzle. But do not know the France mathematician Lucas own this legend, or he inspired by others.

If the legend is true, the monks need $2^{64} - 1$ step to complete this task; if they can complete a disk movement per second, it will need 584.5 billion years to complete. The whole universe is now 13.7 billion years.

There are several variations of this legend: monasteries replaced by monasteries, monks replaced by monks and so on. The location of the monastery is different, one of which is located in Hanoi , Vietnam , it was named "Hanoi tower". There are also background settings such as "gold plate is created by creation" and "monks move every day".

There is indeed a "Buddha" (tower) building in Buddhism; some of the slaves are also covered by the above rules. "Hanoi tower" one may be from the Indochina Peninsula in the colonial period into Europe.

There are three poles A, B, C. There are N (N> 1) perforated discs on the A rod, and the size of the disc is reduced from bottom to top. It is required to move all discs to the C bar according to the following rules:

1. Only one disc can be moved at a time;
2. The market can not be stacked in the small disk above.

Q32. If there are 3 discs on the A rod, how many times do you want to move at least?

Note: The disc can be temporarily placed in the B pole, but also from the A rod out of the disc back to the A rod, but must follow the above two rules.

答案:

Q29. 播放音樂

第一首歌分別在 1:00 , 1:15 , 1:30 , 1:45 播放，周小姐在開始播放前的 3 分鐘內到達便可聽到。

故周小姐在以下時段進入餐廳即可聽到音樂：

1:00 – 1:03, 1:12 – 1:18 , 1:27 – 1:33 和 1:42 – 1:48

$$\therefore \text{概率} = \frac{3\times 7}{60} = 0.3$$

第二首歌分別在 1:03, 1:18, 1:33, 1:48 播放。
故周小姐在以下時段進入餐廳即可聽到音樂：

1:00 – 1:06 , 1:15 – 1:21, 1:30 – 1:36 和 1:45 – 1:51

$$\therefore \text{概率} = \frac{4\times 6}{60} = 0.4$$

The first song starts at 1:00 , 1:15 , 1:30 and 1:45 , If Miss Chow could enter the restaurant within 3 mins before the song starts, she would be able to hear that song.

Therefore Miss Chow has to enter the restaurant within the following time:
1:00 – 1:03, 1:12 – 1:18 , 1:27 – 1:33 and 1:42 – 1:48

$$\therefore \text{probability} = \frac{3\times 7}{60} = 0.3$$

The second song starts at 1:03, 1:18, 1:33, 1:48.
Therefore Miss Chow has to enter the restaurant within the following time:
1:00 – 1:06 , 1:15 – 1:21, 1:30 – 1:36 and 1:45 – 1:51

$$\therefore \text{probability} = \frac{4\times 6}{60} = 0.4$$

Q30. 尤拉公式 (Euler's Formula)

F = 7
V = 10
E = 15

Q31. 殘殺戰俘與死裡逃生

這個問題是比較容易解答的：敵人從 1 號開始，隔一個殺一個，第一圈把奇數號碼的戰士全殺死了。剩下的 32 名戰士需要重新編號，而敵人在第二圈殺死的是重新編排的奇數號碼。

由於第一圈剩下的全部是偶數號 2，4，6，8 …… 64。把它們全部用 2 除，得 1，2，3 …… 32。這是第二圈重新編的號碼。第二圈殺過之後，又把奇數號碼都殺掉了，還剩下 16 個人。如此下去，可以想到最後剩下的必然是 64 號。

$64=2^6$，它可以連續被 2 整除 6 次，是從 1 到 64 中能被 2 整除次數最多的數，因此，最後必然把 64 號剩下。從 $64=2^6$ 還可以看到，是轉過 6 圈之後，才把約瑟夫斯剩下來的。

如果有 65 名戰士被俘，敵人還是按上述方法殘殺戰士，最後剩下的還是 64 號約瑟夫斯嗎？

不是了。因為第一個人被殺後，也就是 1 號被殺後，第二個被殺的必然是 3 號。如果把 1 號排除在外，那麼剩下的仍是 64 個人。對於剩下的 64 個人，新 1 號就應該是原來的 3 號。這樣原來的 2 號就變成新的 64 號了，所以剩下的必然是原來的 2 號。

對於一般情況來說，如果原來有 2^k 個人，最後剩下的必然是 2^k；如果原來有 2^k+1 個人，最後剩下的是 2 號；如果原來有 2^k+2 個人，最後剩下的是 4 號 ……如果原來有 $2^k + m$ 個人，最後剩下的是 2m 號。

比如，原來有 100 人，由於 $100 = 64 + 36 = 2^6+36$，所以最後剩下的是 $2\times36 = 72$ 號；又比如，原來有 111 人，由於 $111 = 64 + 47 = 2^6 + 47$，所以最後剩下的是 $2\times47=94$ 號。

下面把問題改一下：不讓被俘的戰士站成圓圈，而排成一條直線，然後編上號碼。從 1 號開始，隔一個殺一個。殺過一遍之後，再重新編號，從新 1 號開始，再隔一個殺一個。問最後剩下的還是 64 號約瑟夫斯嗎？

答案是肯定的，最後剩下的仍然是約瑟夫斯。

如果戰俘人數是 65 人呢？剩下的還是約瑟夫斯。只要人數不超過 128 人，也就是人數少於 2^7，那麼最後剩下的總是約瑟夫斯。因為從 1 到 128 中間，能被 2 整除次數最多的就是 64。而敵人每次都是殺奇數號留偶數號。所以 64 號總是最後被留下的人。

This question is rather easy. The enemies started killing all the victims with odd numbers in the circle, that is, the 1st, 3rd, 5th and so on. After the first round, there were 32 prisoners still alive and each of them was given a new number in the circle. Once again, all the prisoners with odd numbers were killed.
After the first round, all the prisoners with even numbers were not killed, that is, 2,4, 6, 8…64. We divide these numbers by 2, we get 1, 2, 3…32. After the second round, 16 prisoners were not killed. Hence, the last one who could survive must be number 64.

$64 = 2^6$, it can be divided by 2 consecutively for 6 times. It is also the number from 1 to 64 which is divided by 2 for the most of times. Thus, number 64 must be the one finally remained. From this, we know that Josephus was the one who was not killed after the 6 rounds.

If there were 65 war prisoners and the enemies followed the same way to kill them, would Josephus (No. 64) be the one who could remain alive? The answer is no. When the first one was killed, that is, the one with number one, the second one to be killed was number 3. If we took away number 1, there were still 64 prisoners. For the remaining 64 people, the new number 1 was the one whose original number was 3. Hence, the prisoner whose original number was 2 became the new number 64. As a result, number 2 would be the one who was not killed.

Generally speaking, if there are 2 to the power *k* people, the one who is not killed is the number 2 to the power k. If there are (2 to the power *k* + 1) people, the one who is not killed is number 2. If there are (2 to the power *k* + 2) people, the lucky guy is number 4. Therefore, if there are (2 to the power *k* + *m*) people, the lucky one are number 2m.

For example, there are 100 people. 100 = 64 + 36 = 2 to the power 6 + 36, so the one who is not killed is number $2\times36 = 72$. Another example is when there is 111 people, 111 = 64 + 47 = 2 to the power 6 + 47, so the lucky guy is number 94
($2\times47 = 94$).

Let's modify the question a bit. We ask the war prisoners to form a queue instead of a circle and we give each one of them a number. We kill the prisoners with odd numbers starting from number 1. After the first round of killing, all the prisoners are given a new number starting from 1 again and repeat the same procedure. Will Josephus (number 64) be the one who is not killed in the end?

The answer is yes. The last one left is still Josephus.

If there are 65 prisoners, the one left is still Josephus. If there are not more than 128 people, that is, number of people less than 2 to the power 7, the one left in every round must be Josephus (number 64). It is because from 1 to 128, 64 is the number that can be divided by 2 for most of the times. What's more, the enemies kill prisoners with odd numbers and those with even numbers are not killed. Thus, the one whose number is 64 is always the lucky guy who has the narrow escape.

Q32. 最少要移動多少次才能將 3 個金盤移動到另一根柱子？

$2^3 - 1 = 7$

在真實玩具中，一般盤子數目為 8；最少需移動 255 次。 如果盤子數目為 10，最少需移動 1023 次。 如果盤子數目為 15，最少需移動 32767 次；這就是說，如果一個人從 3 歲到 99 歲，每天移動一塊圓盤，他最多僅能移動 15 塊。如果盤子數目為 20，最少需移動 1048575 次，即超過了一百萬次。

一般而言，可用 $2^n - 1$ 此一般式找出最少移動次數。

In real toys, the general N = 8; at least 255 times to move. If N = 10, move at least 1023 times. If N = 15, at least 32767 times to move; that means that if a person from 3 to 99 years old, every day to move a disk, he can only move up to 15 blocks. If N = 20, at least 1048575 times to move, that is more than a million times.

In general, we can use general form $2^n - 1$ to find the least number of move.

明察秋毫分析題

Q33. 分析題 1

Albert 和 Bernard 剛剛和 Cheryl 成為了朋友，他們想知道 Cheryl 的生日日期，Cheryl 最終給他們十個可能日期：
5 月 15 日、5 月 16 日、5 月 19 日
6 月 17 日、6 月 18 日
7 月 14 日、7 月 16 日
8 月 14 日、8 月 15 日、8 月 17 日

Cheryl 分別告訴了 Albert 她生日的月份 和告訴了 Bernard 她生日的日子。
之後 Albert 就表述： 我不知道 Cheryl 的生日，但我知道 Bernard 也不會知道。

Bernard 回答： 一開始我不知道 Cheryl 的生日，現在我知道了。
Albert 也回答： 那我也知道 Cheryl 的生日了。

究竟， Cheryl 的生日在何時？

Albert and Bernard just become friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates,

May 15 May 16 May 19
June 17 June 18
July 14 July 16
August 14 August 15 August 17

Cheryl then tells Albert and Bernard separately the month and the day of her birthday respectively.

Albert： I don't know when Cheryl's birthday is, but I know that Bernard does not know too.
Bernard： At first I don't know when Cheryl's birthday is, but I know now.
Albert： Then I also know when Cheryl's birthday is.

So when is Cheryl's birthday？

Q34. 分析題 2

小明參加大抽，若能從 8 塊大小、形狀、顏色完全相同中找出真的金幣(較重的一塊便是金幣)，他便可能獲得該金幣作為禮物。現在小明獲分發一個天秤，但只可以秤兩次，你能協助小明把真金幣秤出來嗎？

Siu Ming took part in a lucky draw. If he could find the real gold coin (the heavier one) from 8 pieces of gold coins which were exactly the same size, shape and color, he could get that real gold coin as the prize. Siu Ming was given a balance and he was allowed to weigh twice. Can you help him find the real gold coin?

Q35. 分析題 3

一個藥店購買了 20 瓶藥，每瓶裝有 1000 片藥。後來藥廠來電藥店，說所有的藥品需檢查後方可出售，因為在其中一個瓶中放錯了每片比原來重 10 毫克的藥片。藥劑師非常生氣，要找出錯了的藥瓶非常麻煩，要在每瓶中取出一片藥，秤出它們的重量，要秤 20 次。有什麼方法可以只秤一次便能找出錯了的藥瓶呢？

A pharmacy bought 20 bottles of pills and each bottle had 1000 pills. Later, the pharmaceutical company phoned the pharmacy and said that all the pills needed to be inspected before selling them as they had put the wrong pills in one of the bottles. Each pill in that bottle weighed 10mg heavier than the pills in the other bottles. The pharmacist was furious after knowing this. It would be very troublesome to find out the bottle with the wrong pills since he had to take one pill from each bottle and weighed them one by one which meant that he might need to weigh 20 times. Can you think of a method that he only had to weigh once to find out which bottle contained the wrong kind of pills?

Q36. 分析題 4

一位年輕人到商店購買禮物。這禮物的成本是 28 元，售價是 31 元。這年輕人掏出 100 元購買這件禮物，但因老闆沒有零錢，便向街坊換了 100 元零錢，並找回 69 元給年輕人。後來街坊發現該 100 元是假鈔，老闆無奈把 100 元還給街坊。問是次商店老闆共損失多少？

A young man went to buy a present one day. The cost of the present was $28 and the selling price was $31. This young man paid it with a $100 note, but the shopkeeper did not have enough change. Therefore, he went to look for change from his neighbor. When he came back, he gave the change to that young man. However, his neighbor found that the $100 note was a counterfeit. If the boss needed to return $100 to the neighbor, how much loss did the shopkeeper suffer altogether?

明察秋毫分析題

答案

Q33. 分析題 1

在出現的十個日子中，只有 18 日和 19 日出現過一次，如果 Cheryl 生日日子是 18 或 19 日，那知道日子的 Bernald 就能猜到月份，一定知道 Cheryl 的生日是何月何日。

為何 Albert 肯定 Bernald 不知道 Cheryl 的生日呢？如上述，因為 5 月和 6 月均有只出現過一次的日子 18 日和 19 日，知道月份的 Albert 就能判斷，到底 Bernald 有沒有肯定的把握，所以她的生日一定是 7 月或 8 月。

Bernald 的說話也提供訊息，因為在 7 月和 8 月剩下的 5 個日子中，只有 14 日出現過兩次，如果 Cheryl 告訴 Bernald 她的生日日子是 14 日，那 Bernald 就沒有可能憑 Albert 的一句話，猜到她的生日日子。所以有可能的日子，只剩下 7 月 16 日、8 月 15 日和 8 月 17 日。

在 Bernald 說話後，Albert 也知道了 Cheryl 的生日，反映 Cheryl 的生日月份不可能在 8 月，因為 8 月有兩個可能的日子，7 月卻只有一個可能性。

所以答案是 7 月 16 日。

Albert is told either May, June, July or August.
Bernard is told either 14, 15, 16, 17, 18 or 19
Let's go through it line by line.

Albert: I don't know when Cheryl's birthday is, but I know that Bernard doesn't know too.

All Albert knows is the month, and every month has more than one possible date, so of course he doesn't know when her birthday is. The first part of the sentence is redundant.

The only way that Bernard could know the date with a single number, however, would be if Cheryl had told him 18 or 19, since of the ten date options only these numbers appear once, as May 19 and June 18.

For Albert to know that Bernard does not know, Albert must therefore have been told July or August, since this rules out Bernard being told 18 or 19.

Line 2) Bernard: At first I don't know when Cheryl's birthday is, but now I know.

Bernard has deduced that Albert has either August or July. If he knows the full date, he must have been told 15, 16 or 17, since if he had been told 14 he would be none the wiser about whether the month was August or July. Each of 15, 16 and 17 only refers to one specific month, but 14 could be either month.

Line 3) Albert: Then I also know when Cheryl's birthday is.

Albert has therefore deduced that the possible dates are July 16, Aug 15 and Aug 17. For him to now know, he must have been told July. Since if he had been told August, he would not know which date for certain is the birthday.
The answer, therefore is July 16.

Q34. 分析題 2

分四組，A(3 塊)、B(3 塊)、C(1 塊)、D(1 塊)
秤第 1 次： A 組與 B 組對秤一次
秤第 2 次： 若第 1 次相同，則 C 組及 D 組對秤一次，重者便是真的。
若第 1 次不同，則把較重的一組再分 2 組【E 組(2 塊)、F 組(1 塊)】，把 E 組兩塊金幣對秤。若相同，即 F 為真；若不同，則較重為真。

Divide the 8 gold coins into 4 groups: A (3 pieces), B (3 pieces), C (1 piece), D(1 piece)

The first weighing: Put A and B on the balance
The second weighing: If A and B are the same weight, then put C and D on the balance.

The heavier one is the real gold coin. If A and B are different weights, take the heavier group and further divide it into 2 groups (E (2 pieces) and F (1 piece)). Then put E on the balance. If the 2 coins are the same weight, F is the real one. If they are different weights, the heavier is the real one.

Q35. 分析題 3

按藥瓶位置為每一瓶逐一編上號碼(由 1 開始)，然後按瓶的編號拿出藥丸數目，即第 1 瓶拿 1 粒藥，第 2 瓶藥拿 2 粒，如此類推。然後一併秤出結果。若較原來多 10 毫克，即第 1 瓶為錯，多 20 毫克，即第 2 瓶為錯，如此類推。

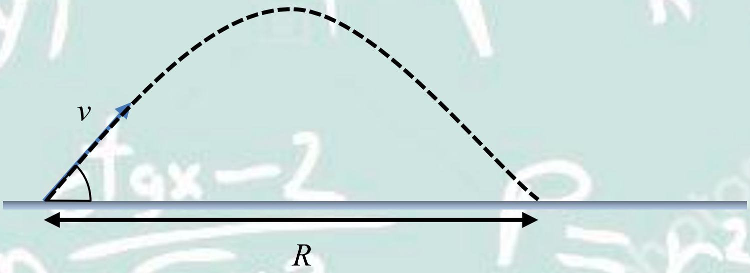
We label the 20 bottles from 1 to 20. Next, take 1 pill from the first bottle, 2 pills from the second bottle, 3 pills from the third bottle and up to 20 pills from the 20th bottle. After that, we weigh all the pills together. If the total weight is 10mg heavier than the expected weight, then the first bottle contains the wrong pills. If the total weight is 20mg heavier than the expected weight, then the second bottle contains the wrong pills and so on.

Q36. 分析題 4.

損失的金錢 = 成本 + 找續的金額 = $97

Loss = Cost + Change = $97

Q37. 炮彈的射程



如一物件以初速 v 拋出，物件會沿拋物線運動。已知物件的射程 R ，可用以下公式計算：

$$R = \frac{u^2 \sin 2\theta}{g}$$

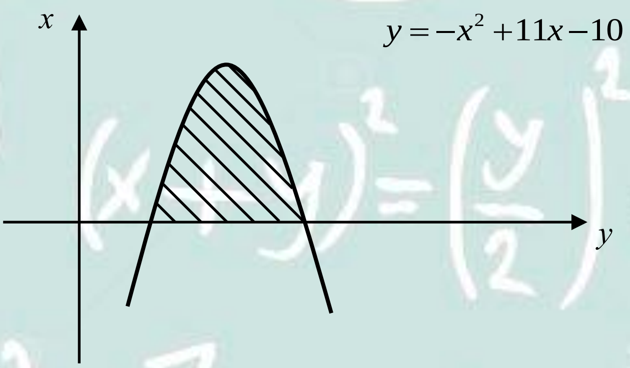
當中 θ 為初速 u 與地面的夾角。g 為常數。
θ 要是多少才能使 R 達最大值？

When we project an object with initial velocity v, the object will follow a parabolic path. The range (R) can be calculated by using the following equation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

Where θ is the angle between initial velocity v and horizontal. g is a constant.
What is the value of θ when R is at maximum?

Q38. 估算陰影面積



以下哪個是上圖陰影面積可能的值？

A. 75.5　　B. 91.125
C. 115.125　　D. 121.5

Which of the following is a possible value of area of the shaded region in the above figure?

A. 75.5　　B. 91.125
C. 115.125　　D. 121.5

Q39.

設 $F = 1 + 2 + 2^2 + 2^3 + 2^4 ... + 2^{99}$ 和 $T = \sqrt{\dfrac{\log(1+F)}{\log 2}}$ ，求 T 的值。

Let $F = 1 + 2 + 2^2 + 2^3 + 2^4 ... + 2^{99}$ and $T = \sqrt{\dfrac{\log(1+F)}{\log 2}}$, find the value of T.

Q40.

設 $abcd = 1$。若

$$K = \frac{1}{1+a+ab+abc} + \frac{1}{1+b+bc+bcd} + \frac{1}{1+c+cd+cda} + \frac{1}{1+d+da+dab} ,$$

求 K 的值。

Let $abcd = 1$. If

$$K = \frac{1}{1+a+ab+abc} + \frac{1}{1+b+bc+bcd} + \frac{1}{1+c+cd+cda} + \frac{1}{1+d+da+dab} ,$$

find the value of K.

將軍級數的數學

答案

Q37. 炮彈的射程

$\because R = \dfrac{u^2 \sin 2\theta}{g}$

其中 u 和 g 是常數。

$\therefore R \propto \sin 2\theta$

當 $2\theta = 90^\circ$ 時， $\sin 2\theta$ 達到最大值，即 R 亦達其最大值。

$\therefore$ 當 $\theta = 45^\circ$ °時，R亦達其最大值。

$\because R = \dfrac{u^2 \sin 2\theta}{g}$

where u and g are constants.

$\therefore R \propto \sin 2\theta$

When $2\theta = 90^\circ$, $\sin 2\theta$ attains maximum value, therefore R attains maximum value.

$\therefore$ When $\theta = 45^\circ$, R attains maximum value.

Q38. $y = -x^2 + 11x - 10$

代 (put) $y = 0$

$0 = -x^2 + 11x - 10$

$x = 1$ 或 (or) $x = 10$

面積 (area) $= \int_1^{10} (-x^2 + 11x - 10)dx$

$= 121.5$

$=$ D

Q39. $F = 1 + 2 + 2^2 + 2^3 + 2^4 ... + 2^{99}$

$F = \dfrac{1(2^{100} - 1)}{2 - 1}$

$F = 2^{100} - 1$

$T = \sqrt{\dfrac{\log(1 + 2^{100} - 1)}{\log 2}}$

$T = \sqrt{\dfrac{\log(2^{100})}{\log 2}}$

$T = \sqrt{\dfrac{100 \log 2}{\log 2}}$

$T = 10$

Q40.

$$K = \frac{1}{1+a+ab+abc} + \frac{1}{1+b+bc+bcd} + \frac{1}{1+c+cd+cda} + \frac{1}{1+d+da+dab}$$

$$= \frac{d}{d+ad+abd+abcd} + \frac{1}{1+b+bc+bcd} + \frac{b}{b+bc+bcd+abcd} + \frac{1}{1+d+da+dab}$$

$$= \frac{d}{d+ad+abd+1} + \frac{1}{1+b+bc+bcd} + \frac{b}{b+bc+bcd+1} + \frac{1}{1+d+da+dab}$$

$$= \frac{1+b}{1+b+bc+bcd} + \frac{1+d}{1+d+da+dab}$$

$$= \frac{a(1+b)}{a+ab+abc+abcd} + \frac{abc(1+d)}{abc+abcd+abcda+ababcd}$$

$$= \frac{a+ab}{a+ab+abc+1} + \frac{abc+abcd}{abc+1+a+ab}$$

$$= \frac{a+ab+abc+1}{1+a+ab+abc}$$

$$= 1$$

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